Oxford Physics Aptitude Test Specimen May 2009 Answers

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1 Part A

1.1 Question 1

$$\sqrt{(5p-4q)^2-(4p-5q)^2} \tag{1}$$

Start by multiplying out the brackets, do not introduce $\sqrt{3}$ and $\sqrt{2}$ yet as this will overcomplicate the algebra

$$\sqrt{25p^2-40pq+16q^2-16p^2+40pq-25q^2} \tag{2}$$

Cancel out $-40pq$ and $+40pq$ and simplify p^2 and q^2 factors

$$\sqrt{9p^2-9q^2} \tag{3}$$

Take a factor of 9 out

$$\sqrt{9(p^2-q^2)} = \sqrt{9}\sqrt{p^2-q^2} = 3\sqrt{(p^2-q^2)} \tag{4}$$

Now add in values for $p=3$ and $q=2$

$$3\sqrt{3-2} = \pm 3 \tag{5}$$

1.2 Question 2

The question asks for the set of real numbers for which the equation has real, distinct roots. The first thing you should be thinking here is to find the roots using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \tag{6}$$

So...

$$x = \frac{-(\lambda - 3) \pm \sqrt{(\lambda - 3)^2 - 4 \times 1 \times \lambda}}{2} \tag{7}$$

Multiplying out the $(\lambda - 3)^2$ bracket and applying the minus sign at the start

$$\begin{aligned} x &= \frac{3 - \lambda \pm \sqrt{\lambda^2 - 6\lambda + 9 - 4\lambda}}{2} \qquad (8) \\ &= \frac{3 - \lambda \pm \sqrt{\lambda^2 - 10\lambda + 9}}{2} \qquad (9) \end{aligned}$$

Now consider in detail what the question requires. "Real" implies that the number contained within the square root must not be negative (otherwise this would introduce imaginary numbers). "Distinct" implies that the number contained within the square root must not be zero, as this would give two roots which were the same: $(-b \pm 0)/2a$. To find out what values λ can take we apply the quadratic formula again to the contents of the square root i.e. to

$$\lambda^2 - 10\lambda + 9 = 0 \tag{10}$$

So

$$\begin{aligned} \lambda &= \frac{10 \pm \sqrt{(-10)^2 - 4 \times 1 \times 9}}{2} \qquad (11) \\ &= \frac{10 \pm 8}{2} \qquad (12) \\ & \qquad (13) \end{aligned}$$

This has solutions of 1 and 9. These are boundary values. We now need to identify which side of these numbers (1 and 9) allows our equation for λ to have a positive value. Considering the "u" shape of a quadratic with a positive square term (as opposed to the "n" shape of a quadratic with a negative square term), the quadratic for lambda will be greater than zero if lambda is away from the middle i.e. less than 1 or greater than 9. So

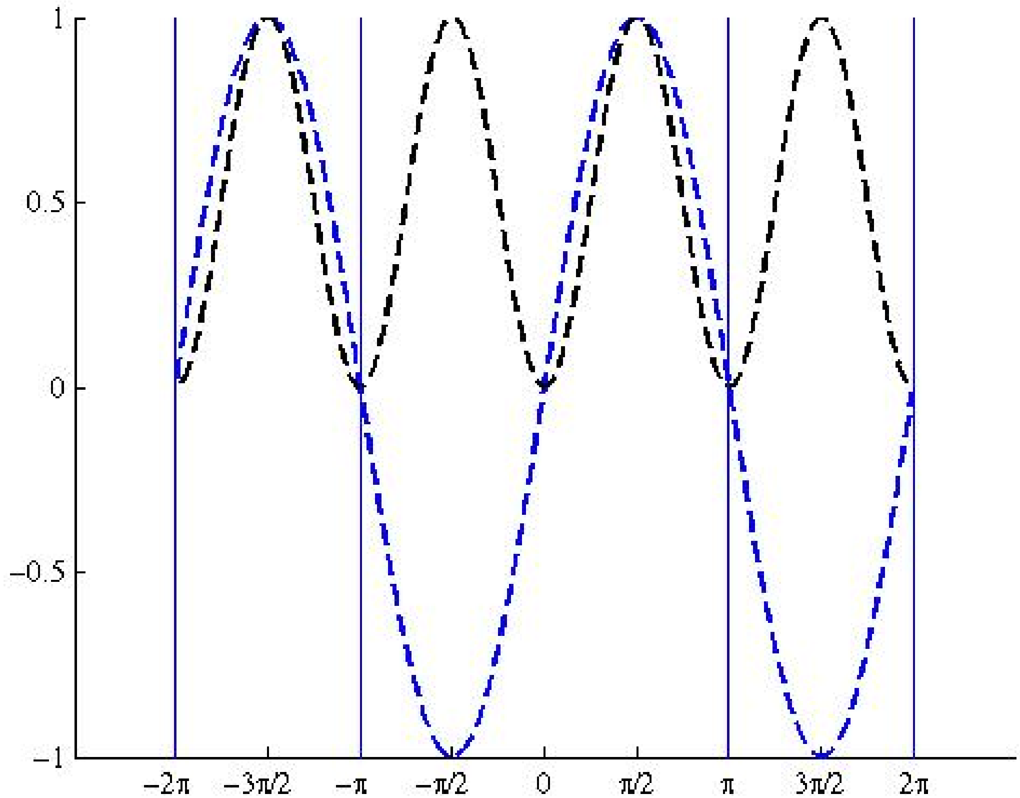
$$\lambda < 1 \tag{14}$$

and

$$\lambda > 9 \tag{15}$$

1.3 Question 3

1.3.1 part i



1.3.2 part ii

Here you need to think in detail about the values $\sin x$ and $\tan x$ can take. Remember that

$$\sin x = \frac{opposite}{hypotenuse} \quad (16)$$

$$\tan x = \frac{opposite}{adjacent} \quad (17)$$

Now if $0 < x < \pi/2$ you are just considering the angles allowed in a triangle. You also know that the hypotenuse of a triangle must be the longest side. Looking closely at the formulae for sin and tan shows that the numerator is same. If the denominator is larger (as is the case for sin), the fraction is smaller. Therefore $\sin x$ is smaller than $\tan x$

1.3.3 part iii

Firstly recognise that $\cos^4\theta = (\cos^2\theta)^2$ so that

$$\begin{aligned} \cos^4\theta &= \frac{1}{4}(1+\cos 2\theta)^2 && (18) \\ 4\cos^4\theta &= 1 + 2\cos 2\theta + \cos^2 2\theta && (19) \end{aligned}$$

Now reapply the given equality to find an equality for $\cos^2 2\theta$

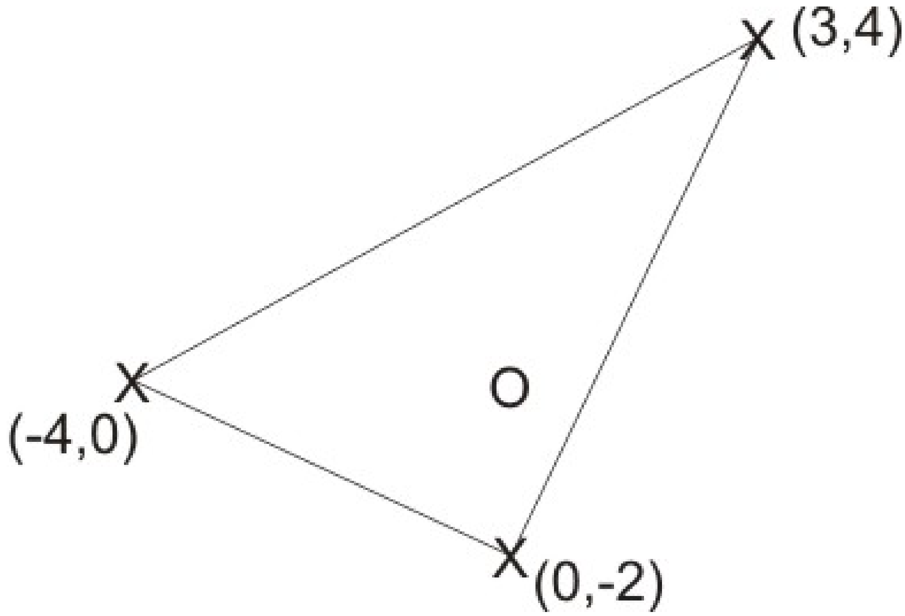
$$\cos^2 2\theta = \frac{1}{2}(1+\cos 4\theta) \qquad (20)$$

Substitute this into the previous equation

$$\begin{aligned} \cos^4\theta &= \frac{1}{4}\left(1 + 2\cos 2\theta + \frac{1}{2}(1+\cos 4\theta)\right) && (21) \\ &= \frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{8} + \frac{1}{8}\cos 4\theta && (22) \\ &= \frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta && (23) \\ &= \frac{1}{8}(3 + 4\cos 2\theta + \cos 4\theta) && (24) \end{aligned}$$

1.4 Question 4

First start with a quick sketch to check orientation and shape of the triangle.



Three points, one of which is not on the straight line joining the other two defines a triangle. A right angle triangle has an angle of 90° between two of the sides. This can be shown by calculating the angles between sides.

A more elegant way is to remember than the gradients of two perpendicular lines multiply to -1. So

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{3 - 0} = 2 \tag{25}$$

and

$$m_2 = \frac{y_3 - y_1}{x_3 - x_1} = \frac{0 - (-2)}{-4 - 0} = -\frac{1}{2} \tag{26}$$

$$m_1 \times m_2 = 2 \times -\frac{1}{2} = -1 \tag{27}$$

Therefore these lines are perpendicular and make up a triangle.

The area is given by the usual equation $A = 0.5 \times base \times height$ so we need to find the lengths of the sides using Pythagoras. Staring with the bottom right edge

$$a = \sqrt{6^2 + 3^2} = \sqrt{45} \tag{28}$$

and the bottom left edge

$$a = \sqrt{4^2 + 2^2} = \sqrt{20} \tag{29}$$

Therefore

$$Area = \frac{1}{2} \times \sqrt{45} \times \sqrt{20} = \frac{1}{2}\sqrt{45 \times 20} = \frac{1}{2}\sqrt{900} = \frac{1}{2} \times 30 = 15 \qquad (30)$$

1.5 Question 5

For this question you need to remember the definition of logs

$$\begin{aligned} a &= b^c && (31) \\ c &= \log_c a && (32) \end{aligned}$$

1.5.1 part 1

$$x = 2^2 = 4 \qquad (33)$$

1.5.2 part ii

$$\begin{aligned} 2 &= x^2 && (34) \\ x &= \sqrt{2} && (35) \end{aligned}$$

1.5.3 part iii

$$\begin{aligned} 2 &= 2^x && (36) \\ x &= 1 && (37) \end{aligned}$$

1.6 Question 6

This could be attempted using a Binomial expansion on $(2 + 0.002)^6$, but an easier method is multiply the bracket out in stages.

$$\begin{aligned} (2.002)^6 &= (2.002^3)^2 && (38) \\ &= \left(\left(2 + \frac{2}{1000}\right)^3\right)^2 && (39) \\ &= \left(2^3 + 3 \times 2^2 \times \frac{2}{10^3} + 3 \times 2 \times \frac{2^2}{10^6} + \frac{2^3}{10^9}\right)^2 && (40) \end{aligned}$$

We can ignore the $2^3/10^9$ part as it won't affect the 4th decimal place. So

$$= \left(8 + \frac{24}{10^3} + \frac{24}{10^6}\right)^2 \tag{41}$$
$$= 8^2 + \frac{24^2}{10^6} + \frac{24^2}{10^{12}} + \frac{2 \times 8 \times 24}{10^3} + \frac{2 \times 8 \times 24}{10^6} + \frac{2 \times 24 \times 24}{10^9} \tag{42}$$

We can now ignore the $24^2/10^{12}$ and $2 \times 24 \times 24/10^9$ part as these will not affect the 4th decimal place. So

$$= 64 + \frac{576}{10^6} + \frac{384}{10^3} + \frac{384}{10^6} \tag{43}$$
$$= 64 + \frac{384}{10^3} + \frac{960}{10^6} \tag{44}$$
$$= 64 + 0.384 + 0.00096 \tag{45}$$
$$= 64.38496 \tag{46}$$
$$= 64.3850 \tag{47}$$

1.7 Question 7

You may find it useful to draw a diagram. After first bounce, ball will have traveled a distance of h, on second bounce, ball will have traveled up to $h/3$ and back down again: so a total of $h + 2h/3$. On the third bounce the ball will have traveled an additional distance of $2h/3^2$ and so on. So ignoring the first term of h, there is a geometric series. The sum to infinity is given be

$$\Sigma\infty = \frac{a}{1-r} \tag{48}$$

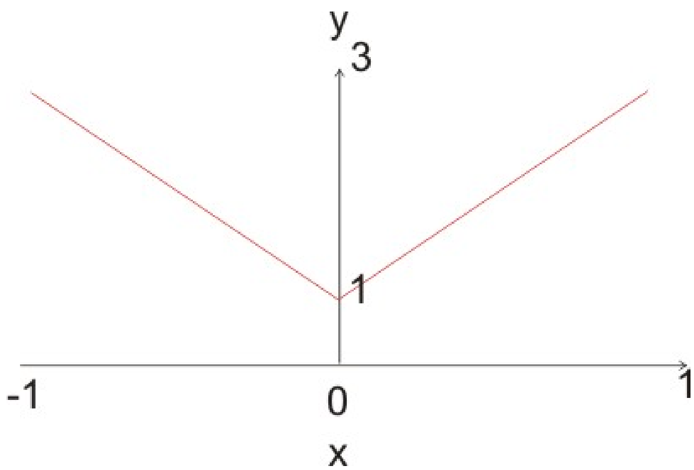
where a is the first term and r is the common ratio. Be careful to include the correct first term here - it's the first term of the geometric series $2h/3$ and not the overall first term h. If H is the total distance traveled

$$H = h + \frac{\frac{2h}{3}}{1 - \frac{1}{3}} = h + \frac{2h}{3}\frac{3}{2} = 2h \tag{49}$$

1.8 Question 8

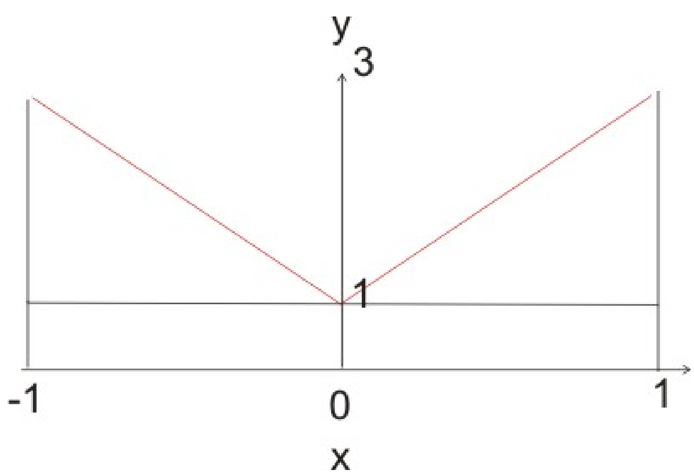
1.8.1 part i

The $|x|$ or modulus of x means we always take a positive value for x. To sketch the graph between -1 and 1, evaluate y at some critical points - say x=-1,0,1. This gives y=3,1,3.



1.8.2 part ii

The area can be calculated by dividing the area into a rectangle and two triangles (note height is 2).



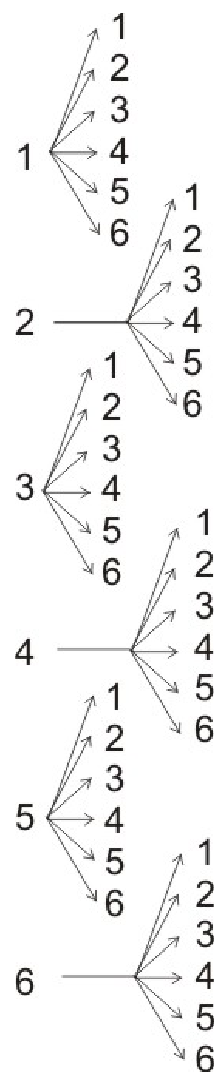
$$Area = 2 \times Triangle + Rectangle \tag{50}$$

$$Area = 2 \times \left(\frac{1}{2} \times 1 \times 2\right) + 2 \times 1 \tag{51}$$

$$= 4 \tag{52}$$

1.9 Question 9

With these type of probability questions, the simplest and safest way is to draw, or at least imagine a tree diagram. This doesn't need to be perfectly neat and tidy (unless a tree diagram is specifically asked for). In this case each branch has probability 1/6



1.9.1 part i

We can easily see the combinations which add to 6 are (1,5),(2,4),(3,3),(4,2) and (5,1) each with probability $1/6 \times 1/6 = 1/36$. The probability of getting a total of 6 between the two dice is 5/36.

1.9.2 part ii

Again you should refer to a tree diagram and remember that we multiply along branches and add between them. So if we get 1 on the first die (with probability 1/6) the probability that the second die shows a number greater than 1 is 5/6. If 2 is thrown first (with probability 1/6) then the second die shows a number greater than 2 with probability 4/6 etc. So

$$\begin{aligned} p &= \frac{1}{6}\frac{5}{6} + \frac{1}{6}\frac{4}{6} + \frac{1}{6}\frac{3}{6} + \frac{1}{6}\frac{2}{6} + \frac{1}{6}\frac{1}{6} && (53) \\ &= \frac{5+4+3+2+1}{36} && (54) \\ &= \frac{15}{36} && (55) \\ &= \frac{5}{12} && (56) \end{aligned}$$

1.10 Question 10

A geometric progression has the form

$$a, ar, ar^2, ar^3, ar^4, ... \quad (57)$$

and an arithmetic progression has the form

$$b, b+c, b+2c, b+3c, b+4c, ... \quad (58)$$

If they have the same first term then a=b. If the second and third terms of the geometric progression are equal to the third and fourth terms of the arithmetic progression respectively

$$\begin{aligned} ar &= a + 2c && (59) \\ ar^2 &= a + 3c && (60) \end{aligned}$$

1.10.1 part i

To find the common ratio we need to eliminate either a or c from these two equations. Choose c, so

$$\begin{aligned}
\frac{ar-a}{2} &= c && (61)\\
\frac{ar^2}{3} &= c && (62)\\
\frac{ar-a}{2} &= \frac{ar^2-a}{3} && (63)\\
3ar-3a &= 2ar^2-2a && (64)\\
3ar &= 2ar^2+a && (65)\\
3r &= 2r^2+1 && (66)\\
2r^2-3r+1 &= =0 && (67)
\end{aligned}$$

Now, apply the quadratic formula to solve for r

$$\begin{aligned}
r &= \frac{3\pm\sqrt{9^2-8}}{4} && (68)\\
r &= \frac{3\pm 1}{4} && (69)\\
r &= 1\text{or}\frac{1}{2} && (70)
\end{aligned}$$

As the question says the second and third terms of the geometric progression are distinct, r must equal 1/2 and not 1.

1.10.2 part ii

The fifth term of the arithmetic progression is $a+4c$. So eliminate r from the original two equations by making it the subject. Firstly divide them

$$\begin{aligned}
\frac{ar^2}{ar} &= \frac{a+3c}{a+2c} && (71)\\
r &= \frac{a+3c}{a+2c} && (72)
\end{aligned}$$

Rearranging the first equation gives

$$r = \frac{a+2c}{a} \qquad (73)$$

Putting these together to eliminate r gives

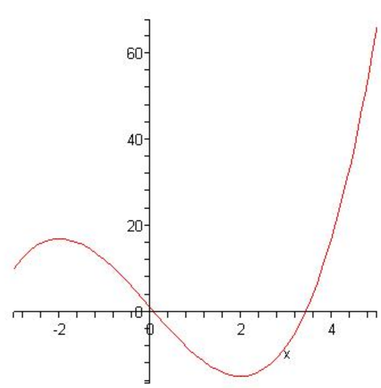
$$\frac{a+3c}{a+2c} = \frac{a+2c}{a} \tag{74}$$
$$a(a+3c) = (a+2c)^2 \tag{75}$$
$$a^2+3ac = a^2+4ac+4c^2 \tag{76}$$
$$0 = ac+4c^2 \tag{77}$$
$$0 = a+4c \tag{78}$$

1.11 Question 11

To find the maximum and minimum values of a cubic equation within a certain interval you should be thinking about differentiating the equation given and setting it equal to zero to find the turning points.

$$y = x^3-12x+2 \tag{79}$$
$$\frac{\delta y}{\delta x} = 3x^2-12=0 \tag{80}$$
$$3x^2 = 12 \tag{81}$$
$$x^2 = 4 \tag{82}$$
$$x = \pm 2 \tag{83}$$

Now consider drawing a sketch. The graph has the usual cubic form with a positive gradient as the x^3 term is positive. Calculate y at a number of 'critical' points - the turning points, the limits of the region we are considering and zero: x=-3,-2,0,2,5. This gives y=10,17,1,-15,66 respectively.



From the sketch is can be clearly seen that the minimum value of x is -15 and the maximum is 66.

2 Part B

2.1 Question 12

With the detector 6cm from the source, no α will be detected since its range in air is $\approx$ 5cm.

The aluminium plate decreases the counts from 74 to 45 per min therefore this must block something. This must be β.

When the source is removed and the count rate stays the same it shows there was nothing getting through the aluminium - the count rate of 45 per min is the background.

If there is no plate and the source is 2cm away, the count rate increases to 5000 per min - this suggests that there is α present. So the answer is C: alpha and beta

2.2 Question 13

If the left engine stops, the forces forwards produced by the two engines will be imbalanced. There will still be the same force on the right, but no forwards force on the left. Therefore the place turns to the left.

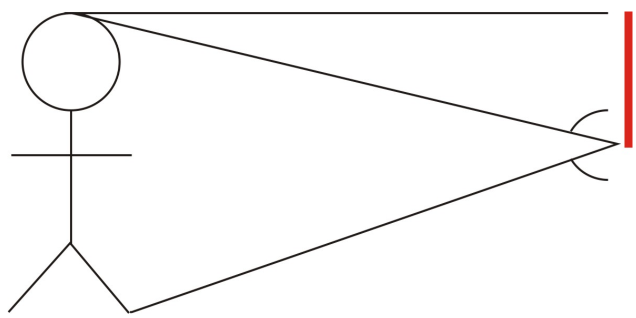
If one engine stops, there will be less force pushing it forwards which will reduced its speed. As a result of the speed decreasing, the lift generated by the wings will be less and the plane will fall. So the answer is C, it turns left, falls and slows.

2.3 Question 14

Terminal velocity is the constant speed reached when the forces of air resistance (upthrust) and weight are equal in size (but opposite in direction). The answer is A.

2.4 Question 15

The answer is A: 1/2 your height. This can be seen by drawing a simple ray diagram. Remember: angle of incidence equals angle of reflection.



2.5 Question 16

All dimensions expand by 1% therefore the radius and circumference of both the hole and the disk expand. The answer is A.

2.6 Question 17

The simplest way to answer this question is to use the equation for the heat capacity of the water:

$$E = mc\Delta T \tag{84}$$

Where E is the energy, m is the mass, c is the specific heat capacity and ΔT is the change in temperature.

$$\begin{aligned}
\text{Energy of 10kg at 15°C} &+ \text{Energy of additional water at 50°C} && (85)\\
&= \text{Energy of total water at 37°C} && (86)\\
& && (87)
\end{aligned}$$

Substituting for $E = mc\Delta T$, we can neglect the c term as it is common to all terms on both sides.

$$\begin{aligned}
10 \times 15 + x \times 50 &= (10 + x) \times 37 && (88)\\
150 + 50x &= 370 + 37x && (89)\\
13x &= 220 && (90)\\
x &= \frac{220}{13} \approx 17 && (91)
\end{aligned}$$

The answer is C: 17kg. Alternatively base yourself at 37°C:

$$\begin{aligned}
10 \times 22 &= m \times 13 && (92)\\
m &= \frac{220}{13} && (93)
\end{aligned}$$

2.7 Question 18

Acceleration will be $10ms^{-2}$ as the acceleration will always be equal to g. The answer is C.

2.8 Question 19

Recall the equation $V = IR$. Two bulbs will have twice as much resistance, the voltage will be the same, so the current will be half. The answer is A: less current to the series combination.

2.9 Question 20

From Earth: A lunar eclipse occurs when the Moon passes behind the Earth such that the Earth blocks the Suns rays from striking the Moon. A solar eclipse occurs when the Moon passes between the Sun and the Earth, and the Moon fully or partially covers the Sun as viewed from some location on Earth.

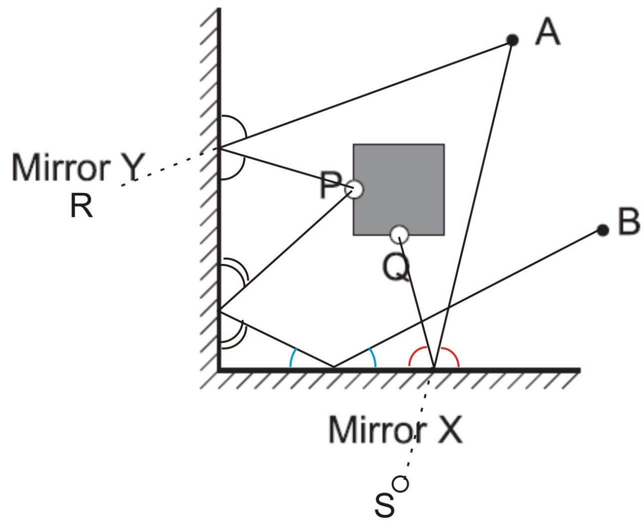
From the Moon a solar eclipse will occur when the Earth passes between the Sun and the Moon, Thus, on Earth a lunar eclipse will occur. The answer is B.

2.10 Question 21

Your image appears behind the mirror - the same distance behind the mirror as you are in front of it. Therefore, if you move 5m towards the mirror, your image also gets 5m closer to you - giving a total of 10m. The answer is C.

2.11 Question 22

Remember: angle of incidence equals angle of reflection.



2.12 Question 23

Demote the slepton by s and the antisleption by $\bar{s}$; the hozon by h and the elephoton by e. From the question write some equations for the charge

$$q_s + q_s + q_h = 0 \quad (94)$$
$$q_s + q_s + q_s + q_h + q_e = 1 \quad (95)$$
$$q_{\bar{s}} + q_e = -2 \quad (96)$$

and mass

$$m_s + m_h + m_e = 6m_e \quad (97)$$
$$m_{\bar{s}} + m_e = 3m_e \quad (98)$$

Since $m_{\bar{s}} = m_s$

$$m_{\bar{s}} + m_e = 3m_e \quad (99)$$
$$m_{\bar{s}} = 2m_e \quad (100)$$
$$m_s = 2m_e \quad (101)$$

Substituting $m_s = 2m_e$ into $m_s + m_h + m_e = 6m_e$ gives

$$m_s + m_h + m_e = 6m_e \quad (102)$$

$$\begin{aligned} 2m_e + m_h + m_e &= 6m_e && (103) \\ m_h &= 3m_e && (104) \end{aligned}$$

Moving onto the charge the above equations can be simplified. Since $q_s + q_s + q_h = 0$ and $q_s + q_s + q_s + q_h + q_e = 1$, $q_s + q_e = 1$. Also $q_{\bar{s}} = -q_s$ so $-q_s + q_e = -2$. Rearranging and substituting

$$\begin{aligned} -q_s + q_e &= -2 && (105) \\ -(1 - q_e) + q_e &= -2 && (106) \\ -1 + q_e + q_e &= -2 && (107) \\ -1 + 2q_e &= -2 && (108) \\ 2q_e &= -1 && (109) \\ q_e &= -\frac{1}{2} && (110) \end{aligned}$$

Again substituting

$$\begin{aligned} q_s + q_e &= 1 && (111) \\ q_s - \frac{1}{2} &= 1 && (112) \\ q_s &= \frac{3}{2} && (113) \end{aligned}$$

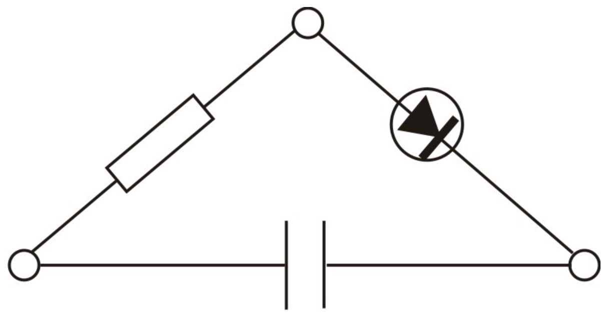
and

$$\begin{aligned} q_s + q_s + q_h &= 0 && (114) \\ \frac{3}{2} + \frac{3}{2} + q_h &= 0 && (115) \\ q_h &= -3 && (116) \end{aligned}$$

2.13 Question 24

2.13.1 part a

As a finite current flows when the battery is connected between A and B this must be the resistor. When the battery is connected one way round across B and C no current flows, then the battery is the other way around a very large current flows. This is characteristic of a diode. By process of elimination, the capacitor must be between A and C. You just need to get the diode around the correct way. Remember current is a flow of electrons, but circuits are drawn according to conventional flow - from positive to negative. The diode symbol has an arrow which points in the direction of conventional flow and so should point from B to C.



2.13.2 part b

$$V = IR \tag{117}$$
$$9 = \frac{3}{1000} \times R \tag{118}$$
$$9 \times \frac{1000}{3} = R \tag{119}$$
$$3000\Omega = R \tag{120}$$

2.13.3 part c

If the battery is connected across the resistor the other way around a current of 3mA will flow through the resistor. Note in a steady state the capacitor doesn't let DC current through as it is effectively an insulator. A further way to look at it is consider that you have a resistor and capacitor in parallel with the battery. The reactance of a capacitor is given by $X_c = 1/2\pi fC$ where f is the frequency. As the frequency tends to zero (for DC), then X_c goes to infinity. Kirchhoff's laws tell us that no current will flow through this branch of the circuit.

2.13.4 part d

Similar to part c: 3mA current will flow. The only additional complication is that you need to consider the diode. In this configuration with C- and

A+ the diode passes current and can effectively be thought of as a simple wire. (Compare with the information given in the question which says a large current flows when C- and B+.)

2.14 Question 25

2.14.1 part a

$$\begin{aligned}
\text{GPE} &= mgh && (121)\\
&= (700 + 600) \times 10 \times 9 && (122)\\
&= 13000 \times 9 && (123)\\
&= 130000 - 13000 && (124)\\
&= 117000J && (125)\\
&= 120kJ && (126)
\end{aligned}$$

$$\begin{aligned}
\text{Power} &= \frac{\text{Energy}}{\text{Time}} && (127)\\
&= \frac{117}{30} && (128)\\
&= 3\frac{27}{30} && (129)\\
&= 3\frac{9}{10} && (130)\\
&= 3.9kW && (131)
\end{aligned}$$

2.14.2 part b

Since the weight is hung from two places, the tension in the wire will be 10,000N. Which effectively acts as a 1000kg counter weight from which you subtract the weight of the lift:

$$1000 - 700 = 300kg \qquad (132)$$

2.14.3 part c

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \qquad (133)$$

$$
\begin{aligned}
&= \frac{9m}{30s} && (134)\\
&= \frac{3}{20} && (135)\\
&= 0.3ms^{-1} && (136)
\end{aligned}
$$

The kinetic energy of the lift and passengers is

$$
\begin{aligned}
\text{KE} &= \frac{1}{2}mv^2 && (137)\\
&= \frac{1}{2} \times 1000 \times 0.3^2 && (138)\\
&= 1000 \times 0.09 && (139)\\
&= 45J && (140)
\end{aligned}
$$

Since the counterweight moves half the distance its speed is half so 0.15ms^{-1}. Its weight is double that of the lift. If the mass is double, but the speed is half, then because of the square factor the final kinetic energy is half or $45/2 = 22.5J$. Thus the total kinetic energy is 67.5J.

2.14.4 part d

Consider the distance traveled by the lift in each of the three stages: acceleration, constant speed, deceleration.

$$
\begin{aligned}
s_{acceleration} &= ut + \frac{1}{2}at^2 = \frac{1}{2} \times a \times 100 = 50a && (141)\\
s_{constant} &= \left(\frac{u+v}{2}\right)t = 10v && (142)\\
s_{deceleration} &= ut + \frac{1}{2}at^2 = 50a && (143)
\end{aligned}
$$

and

$$v = u + at = 10a \qquad (144)$$

Remember that the total distance is 9m so

$$
\begin{aligned}
50a + 10v + 50a &= 9 && (145)\\
100a + 10v &= 9 && (146)\\
100a + 10 \times 10a &= 9 && (147)\\
200a &= 9 && (148)\\
a &= \frac{9}{200}ms^{-2} = 0.045ms^{-2} && (149)
\end{aligned}
$$

Since

$$\begin{aligned} v &= 10a && (150)\\ v &= 10 \times 0.045 && (151)\\ v &= 0.45ms^{-1} && (152) \end{aligned}$$

2.14.5 part e

From part d:

$$a = 0.045ms^{-2} \quad (153)$$

Normally weight is $68 \times 10 = 680N$. When the lift is accelerating upwards change in acceleration is $0.045ms^{-2}$ (from part d). So change in weight is $68 \times 0.045 = 3.06N$. Finally when the lift is traveling at a constant speed the acceleration is just gravity so the answer is the same as part a: 680N.